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Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-98-75

INFORMATION ADVANTAGE IN COURNOT OLIGOPOLY **

Ezra Einy, Diego Moreno and Benyamin Shitovitz *

Abstract

We model an oligopolistic industry where a number of firms that are asymmetrically informed about the environment compete via quantities, and we study how the information available to a firm affects its equilibrium profits. Indeed we find that if all firms have access to the same constant returns to scale technology, in any Bayesian equilibrium the information advantage of a firm is rewarded.

Keywords: Cournot Oligopoly, Asymmetric Information, Information Advantage, Bayesian Equilibrium, Correlated Equilibrium.

* **Einy**, Departament of Economics, Ben Gurion University of the Negev (Beer Sheva, Israel 84105). E-mail: einy@bgumail.bgu.ac.il; **Moreno**, Departamento de Economía, Universidad Carlos III de Madrid (28903 Getafe, Spain). E-mail: dmoreno@eco.uc3m.es; **Shitovitz**, Department of Economics, University of Haifa (Haifa, Israel 31905). E-mail: binya@econ.haifa.ac.il

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1 Introduction

In strategic settings in which agents are asymmetrically informed about the environment, it is natural to ask the question whether an individual's information advantage is rewarded. It is easy to find examples of equilibria, however, in which an individual is worse off than other individuals in spite of the fact that his information is superior (see, e.g. Milgrom and Roberts (1982), and Example 3.4 below). Thus, no general results are to be found. Nevertheless, in some specific economic contexts it has been established that an agent with an information advantage is unambiguously better off.

The issue of information advantage has been a recurrent topic in the literature on auctions. Milgrom (1979), for example, has shown that in a first price auction if Bidder A has an information advantage over Bidder B (i.e., if A's information partition is finer than B's), then in any equilibrium B's expected payoff is zero, and therefore it is less than or equal to A's expected payoff (see also Milgrom and Weber (1982), Theorem 3). In the literature on general equilibrium with differential information, it has been studied how alternative solution concepts treat the information advantage of a trader. Einy, Moreno and Shitovitz (1998), for example, show that, when there is no information exchange, (Radner) competitive equilibria and (private) core allocations reward the information advantage of a trader (see also Koutsougeras and Yannelis (1993)). In this context, Krasa and Yannelis (1994) argue that Shapley (private) value allocations also reward the information advantage of a trader.

In the present paper we model an oligopolistic industry where a number of firms that are asymmetrically informed about the environment compete via quantities, and we study how the information available to a firm affects its equilibrium profits. Indeed we find that in an industry where firms have access to the same constant returns to scale technology, the information advantage of a firm is rewarded; i.e., if Firm A has an information advantage over Firm B, then in any Bayesian equilibrium of the Cournot game with differential information associated to the industry the ex-ante expected profits of Firm A are greater than or equal to the ex-ante expected profits of Firm B.

An interesting corollary of our results is that when there is complete information about the environment, the correlated equilibria of the industry's associated (complete information) Cournot game have the same property. That is, every correlated equilibrium relative to a correlation device for which Firm A has an information advantage over Firm B yields expected profits for Firm A greater than or equeal to those of Firm B.

2 The Model

Consider an oligopolistic industry where a group of firms $N = \{1, \ldots n\}$, $n \geq 2$, compete in the production of a homogeneous good. There is uncertainty about the industry's demand and the firms' costs. This uncertainty is described by a probability space $(\Omega, \mathcal{F}, \mu)$, where Ω is the set of states of nature, $\mathcal{F}$ is a σ-field of subsets of Ω, and μ is a σ-additive probability measure on $(\Omega, \mathcal{F})$. (We interpret μ as the common prior of the firms.) Once the state of nature $\omega \in \Omega$ is realized, the market demand, and the firms's costs (the same for all firms) are determined. Write $p : \Omega \times \Re_+ \to \Re$ for the inverse market demand function, and write $c : \Omega \times \Re_+ \to \Re$ for the firms' cost function. Firms do not observe the state of nature that actually occurs, but may have some information about the state. The information of a firm $i \in N$ is described by a measurable partition $\mathcal{P}_i$ of Ω (i.e., $\mathcal{P}_i$ is a finite or countable family of disjoint sets in $\mathcal{F}$ which have positive probability and their union is Ω). We denote by $\mathcal{F}_i$ the σ-subfield of $\mathcal{F}$ generated by the partition $\mathcal{P}_i$. If ω_0 is the true state of nature, Firm i observes the member of $\mathcal{P}_i$ containing ω_0. An *oligopolistic industry with differential information* is thus described by a collection $I = (N, (\Omega, \mathcal{F}, \mu), p, c, (\mathcal{F}_i)_{i \in N})$.

As in Savage's model (see Savage (1954)), in our model the private information of a firm is described by a partition of the space of states of nature. An alternative approach due to Harsanyi (1967-68), represents agents' private information by a set of types, and takes the set of states of nature to be the cross product of the sets of agents' types. Jackson (1991) has shown that both approaches are equivalent (see also Section 2 in Vorha (1999)).

We now introduce the following standard definition from probability theory. Let

T be a set. A family $\{x_t\}_{t\in T}$ of random variables on Ω is called *uniformly integrable* if

$$\lim_{\alpha\to\infty}\sup_{t\in T}\int_{\{|x_t|\geq\alpha\}}|x_t|\,d\mu = 0.$$

We say that a function $f:\Omega\times\Re_+\to\Re$ is *uniformly integrable* if

(2.1) for all $x\in\Re_+$ the function $f(\cdot,x)$ is $\mathcal{F}$-measurable, and

(2.2) the family $\{f(\cdot,x)\}_{x\in\Re_+}$ of random variables is uniformly integrable.

Throughout the paper we assume that the inverse demand function p and the cost function c of any oligopolistic industry with differential information are uniformly integrable.

Let I be an oligopolistic industry with differential information. The *Bayesian game* associated with I is the collection $G(I)=(N,(\Omega,\mathcal{F},\mu),\Re_+,(\mathcal{F}_i)_{i\in N},(\pi_i)_{i\in N})$, where for each $i\in N$, $\pi_i:\Omega\times\Re^n_+\to\Re$, the profit function of firm i, is given for all $\omega\in\Omega$ and $r=(r_1,\ldots,r_n)\in\Re^n_+$ by

$$\pi_i(\omega,r)=r_ip(\omega,\sum_{j\in N}r_j)-c(\omega,r_i).$$

We refer to $G(I)$ as the *Cournot game with differential information* associated with the industry I.

Let $G(I)$ be a Cournot game with differential information. A random strategy for a firm $i\in N$ is an $\mathcal{F}_i$-measurable function $q_i:\Omega\to\Re_+$ whose first and second moments exist. We denote by S_i the set of all random strategies for firm i, and by S the set $\prod_{j\in N}S_j$ of profiles of random strategies.

Let X be a random variable on $(\Omega,\mathcal{F},\mu)$, and let $\mathcal{G}$ be a σ-subfield of $\mathcal{F}$. We denote by $E(X\mid\mathcal{G})$ the conditional expectation of X with respect to $\mathcal{G}$. Our assumptions on p, c, and on the set of strategies of every firm guarantee that for all $i\in N$ and $q\in S$, $E(\pi_i(\cdot,q(\cdot))\mid\mathcal{F}_i)$ exists.

Let $G(I)$ be a Cournot game with differential information. A *Bayesian equilibrium* is a profile of random strategies $q^*=(q_1^*,\ldots q_n^*)\in S$ such that for every $i\in N$ and every $q_i\in S_i$,

$$E(\pi_i(\cdot,q^*(\cdot))\mid\mathcal{F}_i)(\omega)\geq E(\pi_i(\cdot,(q_i(\cdot),q^*_{-i}(\cdot)))\mid\mathcal{F}_i)(\omega), \tag{2.3}$$

for almost every $\omega \in \Omega$.

Remark 2.1. The equilibrium condition (2.3) requires that at a Bayesian equilibrium every firm maximizes its (interim) conditional expected profit at every state of nature. This condition is equivalent to requiring that each firm maximizes its ex-ante expected profit; i.e., condition (2.3) is equivalent to

$$E(\pi_i(\cdot, q^*(\cdot))) \geq E(\pi_i(\cdot, (q_i(\cdot), q^*_{-i}(\cdot)))), \tag{2.4}$$

for every $i \in N$ and every $q_i \in S_i$. This equivalence is obvious in Harsanyi's model of Bayesian games (see, e.g., Section 6.4 in Fudenberg and Tirole (1991)), where uncertainty is about the players' types. We show that it also holds in our model. Clearly (2.3) implies (2.4). In order to prove the converse, assume, contrary to our claim, that q^* satisfies (2.4) but it does not satisfy (2.3). Then there exists an event $B \in \mathcal{F}$ with $\mu(B) > 0$, a firm $i \in N$ and a strategy $q_i \in S_i$ such that

$$E(\pi_i(\cdot, q^*(\cdot)) \mid \mathcal{F}_i)(\omega) < E(\pi_i(\cdot, (q_i(\cdot), q^*_{-i}(\cdot))) \mid \mathcal{F}_i)(\omega),$$

for every $\omega \in B$. Let $A \in \mathcal{P}_i$ be such that $\mu(A \cap B) > 0$, and define

$$\hat{q}_i(\omega) = \begin{cases} q_i(\omega) & \omega \in A, \\ q^*_i(\omega) & \omega \notin A. \end{cases}$$

Then $\hat{q}_i$ is $\mathcal{F}_i$-measurable, and thus $\hat{q}_i \in S_i$. Now let $\omega_0 \in A \cap B$. Then

$$\begin{aligned} E(\pi_i(\cdot, (\hat{q}_i(\cdot), q^*_{-i}(\cdot)))) &= \int_\Omega \pi_i(\omega, (\hat{q}_i(\omega), q^*_{-i}(\omega))d\mu \\ &= \int_A \pi_i(\omega, (q_i(\omega), q^*_{-i}(\omega)))d\mu + \int_{\Omega\setminus A} \pi_i(\omega, q^*(\omega))d\mu \\ &= \mu(A)E(\pi_i(\cdot, (q_i(\cdot), q^*_{-i}(\cdot))) \mid \mathcal{F}_i)(\omega_0) + \int_{\Omega\setminus A} \pi_i(\omega, q^*(\omega))d\mu \\ &> \mu(A)E(\pi_i(\cdot, q^*(\cdot))) \mid \mathcal{F}_i)(\omega_0) + \int_{\Omega\setminus A} \pi_i(\omega, q^*(\omega))d\mu \\ &= E(\pi_i(\cdot, q^*(\cdot))). \end{aligned}$$

But this contradicts (2.4).

3 Information Advantage in Bayesian Equilibria

Proposition 3.1 and Theorem A establish the main result of this paper: In an oligopolistic industry with differential information where firms compete via quantities, if firms' marginal cost is constant then the information advantage of a firm is rewarded; i.e., if Firm i has better information than Firm j, then in any Bayesian equilibrium the (ex-ante) expected profits of Firm i are greater than or equal to those of Firm j.

The following proposition establishes our result when the inverse demand function is differentiable at every state of nature.

Proposition 3.1. *Let $I = (N, (\Omega, \mathcal{F}, \mu), p, c, (\mathcal{F}_i)_{i \in N})$ be an oligopolistic industry with differential information. Assume that*

(P.1) for all $\omega \in \Omega$, $c(\omega, \cdot)$ is affine on $\Re_+$; and

(P.2) for all $\omega \in \Omega$, $p(\omega, \cdot)$ is non-increasing and differentiable on $\Re_+$, and its derivative p' is uniformly integrable.

Let $i, j \in N$ be any two firms such that $\mathcal{F}_i \supseteq \mathcal{F}_j$ (i.e., Firm i's information partition is at least as fine as that of Firm j), and let q^* be any Bayesian equilibrium of $G(I)$. Then

$$E(\pi_i(\cdot, q^*(\cdot))) \geq E(\pi_j(\cdot, q^*(\cdot))).$$

For the proof of Proposition 3.1 we need the following lemma.

Lemma 3.2. *Let $I = (N, (\Omega, \mathcal{F}, \mu), p, c, (\mathcal{F}_i)_{i \in N})$ be an oligopolistic industry with differential information, and let $q = (q_1, \ldots, q_n) \in S$. Then for all $k \in N$ and all $\omega \in \Omega$ we have*

$$E(p(\cdot, \sum_{i \in N} q_i(\cdot)) \mid \mathcal{F}_k)(\omega) = E(p(\cdot, q_k(\omega) + \sum_{i \in N \setminus \{k\}} q_i(\cdot)) \mid \mathcal{F}_k)(\omega).$$

Proof: Let $k \in N$ and $\omega_0 \in \Omega$. Denote by $A(\omega_0)$ the atom of $\mathcal{F}_k$ containing ω_0 (i.e., $A(\omega_0) \in \mathcal{P}_k$). Since q_k is $\mathcal{F}_k$-measurable, it is constant on $A(\omega_0)$. Now

$$E(p(\cdot, \sum_{i \in N} q_i(\cdot)) | \mathcal{F}_k)(\omega_0) = \frac{1}{\mu(A(\omega_0))} \int_{A(\omega_0)} p(\omega, \sum_{i \in N} q_i(\omega)) d\mu$$

$$= \frac{1}{\mu(A(\omega_0))}\int_{A(\omega_0)} p(\omega, q_k(\omega_0) + \sum_{i\in N\setminus\{k\}} q_i(\omega))d\mu$$

$$= E(p(\cdot, q_k(\omega_0) + \sum_{i\in N\setminus\{k\}} q_i(\cdot)) \mid \mathcal{F}_k)(\omega_0).\square$$

With Lemma 3.2 in hand we can now prove Proposition 3.1.

Proof of Proposition 3.1: Let $I = (N, (\Omega, \mathcal{F}, \mu), p, c, (\mathcal{F}_i)_{i\in N})$ be an oligopolistic industry with differential information satisfying the assumptions of Proposition 3.1, and let $i, j \in N$ be such that $\mathcal{F}_i \supseteq \mathcal{F}_j$. Without loss of generality assume that for all $\omega \in \Omega$, $c(\omega, 0) = 0$, and let $d : \Omega \to \Re_+$ denote the marginal cost function. In order to reduce notation, in the rest of the proof we identify $p(\omega, \cdot)$ with $p(\omega, \cdot) - d(\omega)$.

Let q^* be a Bayesian equilibrium of the Cournot game with differential information $G(I)$. Write $Q^* = \sum_{i\in N} q_i^*$. Since for all $k \in N$ the function q_k^* is $\mathcal{F}_k$-measurable, by Theorem 34.3. of Billingsley (1986) we have

$$E(\pi_k(\cdot, q^*(\cdot)) \mid \mathcal{F}_k) = q_k^* E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k). \tag{3.1}$$

Further, for each $k \in N$, q_k^* maximizes firm $k's$ conditional expected profits (given q_{-k}^*). Hence Lemma 3.2 and the first order conditions for maximization of Firm $k's$ conditional expected profits yield

$$q_k^* E(p'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k) + E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k) = 0. \tag{3.2}$$

For every $k \in N$ let

$$A_k = \{\omega \in \Omega \mid E(p'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k)(\omega) = 0\}.$$

Then $A_k \in \mathcal{F}_k$, and by (3.1) and (3.2) we have

$$E(\pi_k(\cdot, q^*(\cdot)) \mid \mathcal{F}_k)(\omega) = E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k)(\omega) = 0, \tag{3.3}$$

for all $\omega \in A_k$. By (3.2), for all $k \in N$ and all $\omega \in \Omega\setminus A_k$ we have

$$q_k^*(\omega) = \frac{E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k)(\omega)}{E(-p'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k)(\omega)}. \tag{3.4}$$

Therefore (3.1) yields

$$E(\pi_k(\cdot, q^*(\cdot)) \mid \mathcal{F}_k)(\omega) = \frac{\left(E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k)\right)^2(\omega)}{E(-p'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k)(\omega)}, \tag{3.5}$$

for all $k \in N$ and all $\omega \in \Omega \backslash A_k$. Since $\mathcal{F}_i \supseteq \mathcal{F}_j$, by Theorem 34.4. of Billingsley (1986) we have

$$E(Z \mid \mathcal{F}_j) = E\left(E\left(Z \mid \mathcal{F}_i\right) \mid \mathcal{F}_j\right), \tag{3.6}$$

for every integrable random variable Z on Ω.

Let $Y = \sqrt{E(-p'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_i)}$. Then $Y \geq 0$ on Ω and $Y > 0$ on $\Omega \backslash A_i$. For all $\omega \in \Omega$ define

$$X(\omega) = \begin{cases} \frac{E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_i)(\omega)}{Y(\omega)} & \text{if } \omega \in \Omega \backslash A_i, \\ 0 & \text{otherwise.} \end{cases}$$

Since $p'(\omega, Q^*(\omega)) \leq 0$ for all $\omega \in \Omega$, by (3.2), $X \geq 0$ on Ω. Also by (3.3) and the definition of X we have

$$XY = E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_i),$$

and

$$X^2 = E(\pi_i(\cdot, q^*(\cdot)) \mid \mathcal{F}_i).$$

Now (3.6) and the Cauchy-Schwartz inequality yield

$$\begin{aligned}
(E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_j))^2 &= \left(E\left(E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_i\right) \mid \mathcal{F}_j)\right)^2 \\
&= \left(E\left(XY \mid \mathcal{F}_j\right)\right)^2 \\
&\leq E\left(X^2 \mid \mathcal{F}_j\right) E\left(Y^2 \mid \mathcal{F}_j\right) \\
&= E\left(E(\pi_i(\cdot, q^*(\cdot)) \mid \mathcal{F}_i\right) \mid \mathcal{F}_j) \, E\left(E(-p'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_i\right) \mid \mathcal{F}_j) \\
&= E(\pi_i(\cdot, q^*(\cdot)) \mid \mathcal{F}_j) E(-p'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_j).
\end{aligned}$$

Therefore by (3.3) and (3.5) we have

$$E(\pi_j(\cdot, q^*(\cdot)) \mid \mathcal{F}_j) \leq E(\pi_i(\cdot, q^*(\cdot)) \mid \mathcal{F}_j). \tag{3.8}$$

Now, by taking integrals over Ω on both sides of (3.8) we obtain

$$E(\pi_j(\cdot, q^*(\cdot))) \leq E(\pi_i(\cdot, q^*(\cdot))),$$

which establishes the proposition. □

The following generalization of Proposition 3.1 covers the cases in which the inverse demand function has kinks. In particular, it covers the usual linear demand case, where the inverse demand is a function of the form

$$p(\omega, Q) = \begin{cases} a(\omega) - b(\omega)Q & \text{if } a(\omega) - b(\omega)Q \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

for $\omega \in \Omega$.

Theorem A. *Let $I = (N, (\Omega, \mathcal{F}, \mu), p, c, (\mathcal{F}_i)_{i \in N})$ be an oligopolistic industry with differential information, and let $G(I)$ be the corresponding Cournot game with differential information. Assume that*

(A.1) for all $\omega \in \Omega$, $c(\omega, \cdot)$ is affine on $\Re_+$;

(A.2) for all $(\omega, x) \in \Omega \times \Re_+$, we have $p(\omega, x) = \max\{p_1(\omega, x), p_2(\omega, x)\}$, where $p_1, p_2 : \Omega \times \Re_+ \to \Re$ are two uniformly integrable functions satisfying

(A.2.1) for all $\omega \in \Omega$, $p_1(\omega, \cdot)$ and $p_2(\omega, \cdot)$ are non-increasing and differentiable on $\Re_+$, and their derivatives p_1' and p_2' are uniformly integrable; and

(A.2.2) for all $(\omega, x) \in \Omega \times \Re_+$, if $p(\omega, \cdot)$ is not differentiable at x, then $p_1(\omega, x) = p_2(\omega, x)$.

Let $i, j \in N$ be any two firms such that $\mathcal{F}_i \supseteq \mathcal{F}_j$, and let q^* be any Bayesian equilibrium of $G(I)$. Then

$$E(\pi_i(\cdot, q^*(\cdot))) \geq E(\pi_j(\cdot, q^*(\cdot))).$$

For the proof of Theorem A we need the following lemma.

Lemma 3.3. *Let f and g be two real-valued functions that are defined and differentiable on a neighborhood of $x_0 \in \Re_+$, and such that $f(x_0) = g(x_0)$ and $f'(x_0) = g'(x_0) = a$. Then the function $h(x) = \max\{f(x), g(x)\}$ is differentiable at x_0 and $h'(x_0) = a$.*

Proof: Let

$$\underline{a} = \underline{\lim}_{x \to x_0, x > x_0} \frac{h(x) - h(x_0)}{x - x_0},$$

and

$$\bar{a} = \overline{\lim}_{x \to x_0, x > x_0} \frac{h(x) - h(x_0)}{x - x_0}.$$

We show that $\underline{a} = \bar{a} = a$. Since for all $x > x_0$

$$\frac{h(x) - h(x_0)}{x - x_0} = \frac{h(x) - f(x_0)}{x - x_0} \geq \frac{f(x) - f(x_0)}{x - x_0},$$

we must have

$$\underline{a} \geq \lim_{x \to x_0, x > x_0} \frac{f(x) - f(x_0)}{x - x_0} = a.$$

Assume, contrary to our claim, that $\bar{a} > a$. Then there exists a sequence $\{x_n\}_{n=1}^{\infty}$ such that $x_n > x_0$ for all n, $\lim_{n \to \infty} x_n = x_0$, and

$$\lim_{n \to \infty} \frac{h(x_n) - h(x_0)}{x_n - x_0} = \bar{a} > a.$$

Since $h(x_n) = \max\{f(x_n), g(x_n)\}$ for all n, there exists a subsequence $\{y_n\}_{n=1}^{\infty}$ of $\{x_n\}_{n=1}^{\infty}$ such that either $h(y_n) = f(y_n)$ or $h(y_n) = g(y_n)$ for all n. As $f(x_0) = g(x_0)$ and $f'(x_0) = g'(x_0) = a$, we have

$$\lim_{n \to \infty} \frac{h(y_n) - h(x_0)}{y_n - x_0} = a,$$

which contradicts our assumption that $\bar{a} > a$. Thus, h'_+ exists at x_0 and $h'_+(x_0) = a$. In a similar way it can be shown that h'_- exists at x_0 and $h'_-(x_0) = a$. □

We now can prove our main result.

Proof of Theorem A: Let $I = (N, (\Omega, \mathcal{F}, \mu), p, c, (\mathcal{F}_i)_{i \in N})$ be an oligopolistic industry with differential information satisfying the assumptions of Theorem A. As in the proof of Proposition 3.1, we assume, without loss of generality, that for all $\omega \in \Omega$, $c(\omega, 0) = 0$, and we identify $p(\omega, \cdot)$ with $p(\omega, \cdot) - d(\omega)$, where $d : \Omega \to \Re_+$ denotes the marginal cost function. Let $i, j \in N$ be such that $\mathcal{F}_i \supseteq \mathcal{F}_j$, and let q^* be a Bayesian equilibrium of $G(I)$. As before, write $Q^* = \sum_{i \in N} q_i^*$. Define

$$A = \{\omega \in \Omega \mid p_1(\omega, Q^*(\omega)) = p_2(\omega, Q^*(\omega)) \text{ and } p_1'(\omega, Q^*(\omega)) \neq p_2'(\omega, Q^*(\omega))\}.$$

Then by $(A.2.1)$, $A \in \mathcal{F}$. For all $(\omega, x) \in \Omega \times \Re_+$ let

$$\hat{p}_1(\omega, x) = \begin{cases} p_1(\omega, x) & \omega \in A, \\ p(\omega, x) & \omega \notin A, \end{cases}$$

and

$$\hat{p}_2(\omega, x) = \begin{cases} p_2(\omega, x) & \omega \in A, \\ p(\omega, x) & \omega \notin A. \end{cases}$$

Then by $(A.2.1)$, $(A.2.2)$ and Lemma 3.3, $\hat{p}_1(\omega, \cdot)$ and $\hat{p}_2(\omega, \cdot)$ are differentiable at $Q^*(\omega)$, for all $\omega \in \Omega$. Now

$$p(\omega, Q^*(\omega)) = \hat{p}_1(\omega, Q^*(\omega)) = \hat{p}_2(\omega, Q^*(\omega)),$$

for all $\omega \in \Omega$. Therefore for all $k \in N$ we have

$$\begin{aligned} E(\pi_k(\cdot, q^*(\cdot))|\mathcal{F}_k) &= q_k^* E(p(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k) \\ &= q_k^* E(\hat{p}_1(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k) \\ &= q_k^* E(\hat{p}_2(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k). \end{aligned} \tag{3.9}$$

For all $k \in N$, let $Q_k^* = \sum_{l \in N \setminus \{k\}} q_l^*$. For all $(\omega, x) \in \Omega \times \Re_+$ define

$$h_k(\omega, x) = xE(p(\cdot, x + Q_k^*(\cdot)) \mid \mathcal{F}_k)(\omega);$$

define also

$$f_k(\omega, x) = xE(\hat{p}_1(\cdot, x + Q_k^*(\cdot)) \mid \mathcal{F}_k)(\omega),$$

and

$$g_k(\omega, x) = xE(\hat{p}_2(\cdot, x + Q_k^*(\cdot)), \mid \mathcal{F}_k)(\omega).$$

Then by Lemma 3.2 and (3.9) we have

$$h_k(\omega, q_k^*(\omega)) = f_k(\omega, q_k^*(\omega)) = g_k(\omega, q_k^*(\omega)).$$

Now for all $(\omega, x) \in \Omega \times \Re_+$ we have

$$\begin{aligned} h_k(\omega, x) &= xE(\max\{\hat{p}_1(\cdot, x + Q_k^*(\cdot)), \hat{p}_2(\cdot, x + Q_k^*(\cdot))\} \mid \mathcal{F}_k)(\omega) \\ &\geq x \max\{E(\hat{p}_1(\cdot, x + Q_k^*(\cdot)) \mid \mathcal{F}_k)(\omega), E(\hat{p}_2(\cdot, x + Q_k^*(\cdot)) \mid \mathcal{F}_k)(\omega)\} \\ &= \max\{f_k(\omega, x), g_k(\omega, x)\}. \end{aligned}$$

Since q^* is a Bayesian equilibrium of $G(I)$, for all $k \in N$ and $\omega \in \Omega$, the function $h_k(\omega, \cdot)$ attains a maximum at $q_k^*(\omega)$. By the above inequality we have $f_k(\omega, x) \leq h_k(\omega, x)$, for all $(\omega, x) \in \Omega \times \Re_+$. Since $h_k(\omega, q_k^*(\omega)) = f_k(\omega, q_k^*(\omega))$, the function $f_k(\omega, \cdot)$ also attains a maximum at $q_k^*(\omega)$ for all $\omega \in \Omega$. Hence, since for all $\omega \in \Omega$, the function $f_k(\omega, \cdot)$ is differentiable at $q_k^*(\omega)$, we have

$$f_k'(\omega, q_k^*(\omega)) = 0,$$

for all $k \in N$ and $\omega \in \Omega$. Now, Lemma 3.2 yields

$$q_k^* E(\hat{p}_1'(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k) + E(\hat{p}_1(\cdot, Q^*(\cdot)) \mid \mathcal{F}_k) = 0.$$

Since for all $\omega \in \Omega$, the function $\hat{p}_1(\omega, \cdot)$ is non-increasing on $\Re_+$, by (3.9) and the same arguments that were used in the proof of Proposition 3.1 (applied to $\hat{p}_1$ and $\hat{p}_1'$) we obtain

$$E(\pi_i(\cdot, q^*(\cdot))) \geq E(\pi_j(\cdot, q^*(\cdot))). \square$$

The assumption that marginal cost is constant plays a critical role in the proofs of our results. Under this assumption firms' cost can be ignored, and the information advantage, allowing a firm to better accomodate its actions to an uncertain environmet, results in higher profits. When marginal cost is not constant, however, in a Bayesian equilibrium a firm that cannot discern between two states of nature may find it optimal to produce a quantity that is not very different from the average quantity of a better informed firm. As the following example shows, if marginal cost is increasing a firm with worse information may have a "cost advantage" that may upset its "information disadvantage."

Example 3.4. Let $I = (N, (\Omega, \mathcal{F}, \mu), p, c, (\mathcal{F}_i)_{i \in N})$ be an oligopolistic industry with differential information where $N = \{1, 2\}$, $\Omega = \{\omega_1, \omega_2\}$, $\mathcal{F} = 2^\Omega$, $\mu(\{\omega_1\}) = \mu(\{\omega_2\})$, $\mathcal{F}_1 = \mathcal{F}$ (i.e., Firm 1 is completely informed), $\mathcal{F}_2 = \{\emptyset, \Omega\}$ (i.e., Firm 2 is completely uninformed); the market demand function is given by

$$p(\omega_1, Q)) = \begin{cases} 120 - 2Q & \text{if } 0 \leq Q \leq 60, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$p(\omega_2, Q) = \begin{cases} 80 - Q & \text{if } 0 \leq Q \leq 80, \\ 0 & \text{otherwise.} \end{cases}$$

Firms' cost function is $c(\omega, q) = q^2$ for all $(\omega, q) \in \Omega \times \Re_+$. The strategy profile $q^* = (q_1^*, q_2^*)$, where $q_1^*(\omega_1) = \frac{1620}{109}$, $q_1^*(\omega_2) = \frac{1760}{109}$, and $q_2^*(\omega_1) = q_2^*(\omega_2) = \frac{1680}{109}$, is a Bayesian equilibrium of the associated Cournot game with differential information $G(I)$. Direct computation yields

$$E(\pi_1(\cdot, q^*(\cdot))) = 592.05 < 593.89 = E(\pi_2(\cdot, q^*(\cdot))).$$

4 Information Advantage in Correlated Equilibria

In this section we study the consequences of Theorem A for oligopolistic industries with complete information, and we establish that the (correlated) equilibria of the associated Cournot game with complete information have a property analogous to the Bayesian equilibria of the Cournot game with incomplete information; namely, in a correlated equilibrium relative to a correlation device for which a Firm A has an information advantage over Firm B, the profits of Firm A are greater than or equal to the profits of Firm B.

An *n-person non-cooperative game with complete information* (or simply a *game*) is a collection $\Gamma = (N, A_1, \ldots, A_n, u_1, \ldots, u_n)$, where $N = \{1, \ldots, n\}$ is the set of players, and for each $i \in N$, A_i and $u_i : \prod_{j=1}^{n} A_j \to \Re$ are, respectively, the set of actions and the payoff function of Player i. Let Γ be a game; a *correlation device for* Γ is a collection $D = ((\Omega, \mathcal{F}, \mu), (\mathcal{P}_i)_{i=1}^{n})$, where $(\Omega, \mathcal{F}, \mu)$ is a probability space, and for all $i \in N$, $\mathcal{P}_i$ is a measurable partition of Ω. For every $i \in N$ let $\mathcal{F}_i$ be the σ-subfield of $\mathcal{F}$ generated by $\mathcal{P}_i$, and let S_i be the set of all $\mathcal{F}_i$-measurable functions from Ω to A_i. Write $S = \prod_{j \in N} S_j$.

Let Γ be a game, and let $D = ((\Omega, \mathcal{F}, \mu), (\mathcal{P}_i)_{i \in N})$ be a correlation device for Γ; a *correlated equilibrium of* Γ *relative to* D is an n-tuple $s^* = (s_1^*, \ldots, s_n^*)$ such that for all $i \in N$ and all $s_i \in S_i$ we have

$$E(u_i(s^*(\cdot))) \geq E(u_i(s_i(\cdot), s_{-i}^*(\cdot))).$$

The notion of correlated equilibrium was introduced in Aumann (1974). A correlated equilibrium can be interpreted as the outcome of Bayesian rationality (see Aumann (1987)). Several equivalent formulations have been suggested (see Aumann (1974, 1987), and Section 2.2 in Fudenberg and Tirole (1991)). The set of correlated equilibria of linear Cournot oligopoly is studied in Liu (1995).

An *oligopolistic industry with complete information* is a collection $I = (N, p, c)$, where $N = \{1, \ldots, n\}$ is the set of firms, $p : \Re_+ \to \Re$ is the inverse demand function and $c : \Re_+ \to \Re$ is the cost function of every firm. The *Cournot game with complete information* associated to an oligopolistic industry with complete information $I =$

(N, p, c) is defined by $\Gamma(I) = (N, \Re_+, \ldots, \Re_+, \pi_1, \ldots, \pi_n)$, where for $i \in N$, the profit function of Firm i, $\pi_i : \Re^n_+ \to \Re$ is given for $r \in \Re^n_+$ by

$$\pi_i(r) = r_i p(\sum_{j \in N} r_j) - c(r_i).$$

Theorem B. *Let $I = (N, p, c)$ be an oligopolistic industry with complete information, and let $\Gamma(I)$ be the corresponding Cournot game with complete information. Assume that*

$(B.1)$ $c(\cdot)$ is affine on $\Re_+$;

$(B.2)$ for all $x \in \Re_+$, $p(x) = \max\{p_1(x), p_2(x)\}$, where $p_1, p_2 : \Re_+ \to \Re$ satisfy

$(B.2.1)$ p_1 and p_2 are differentiable and non-increasing on $\Re_+$; and

$(B.2.2)$ for all $x \in \Re_+$, if p is not differentiable at x, then $p_1(x) = p_2(x)$.

Let s^* be a correlated equilibrium of $\Gamma(I)$ relative to a correlation device $D = ((\Omega, \mathcal{F}, \mu), (\mathcal{P}_i)_{i=1}^n)$, and let $i, j \in N$. If $\mathcal{P}_i$ is at least as fine as $\mathcal{P}_j$, then

$$E(\pi_i(s^*(\cdot))) \geq E(\pi_j(s^*(\cdot))).$$

Proof: Let $I = (N, p, c)$ be an oligopolistic industry with complete information satisfying the assumptions of Theorem B, and let s^* be a correlated equilibrium of the associated Cournot game with complete information $\Gamma(I)$ relative to a correlation device $D = ((\Omega, \mathcal{F}, \mu), (\mathcal{P}_i)_{i=1}^n)$. For each $i \in N$ let $\mathcal{F}_i$ be the σ-subfield of $\mathcal{F}$ generated by $\mathcal{P}_i$. Consider the Cournot game with differential information $G(I) = (N, (\Omega, \mathcal{F}, \mu), \Re_+, (\mathcal{F}_i)_{i \in N}, (\pi_i)_{i \in N})$. (Note that the payoff functions π_i are not state dependent.) By Remark 2.1, any correlated equilibrium s^* of $\Gamma(I)$ is a Bayesian equilibrium of $G(I)$. Let $i, j \in N$ be two firms such that $\mathcal{P}_i$ is at least as fine as $\mathcal{P}_j$. By Theorem A we have

$$E(\pi_i(s^*(\cdot))) \geq E(\pi_j(s^*(\cdot))). \ \Box$$

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